# Fluctuational attenuation of sound in smectics

E V Gurovich, E I Kats and V V Lebedev

L D Landau Institute for Theoretical Physics, Moscow, USSR

**Abstract.** The low-frequency dynamics of smectics C is studied. A system of non-linear equations is given, on the basis of which a spectrum of eigenmodes of smectic C is obtained. An estimate is made of the coefficients entering the dispersion laws and a description of their critical behaviour near A–C transition is given. The analysis shows that orientation and undulation modes always couple strongly, which results in an essential difference of the undulation dispersion law in smectic C compared to smectic A or B. Owing to this the coefficients at $\omega^{-1}$ in anomalous fluctuational contributions to the viscosities of smectics A and C differ. The results of this paper make it possible to give a complete interpretation of the experimental data on sound attenuation in smectics.

## 1. Introduction

Smectics are liquid–crystalline phases with a pronounced layered structure. One can experimentally observe various smectic phases labelled as smectics A–I. It should be borne in mind that a large number of smectic phases are layered 3D crystals. The genuine smectics (where the shear modulus is zero) are only smectics A, C and some smectics B. The latter are usually called hexatics to be distinguished from crystalline smectics B.

In this paper we shall study one of the most common smectic phases, namely, the C phase. In contrast to the A phase, where the director is oriented perpendicular to the smectic layers, the director of the C phase is tilted to the smectic layers. As a rule, the smectic C phase emerges from the smectic A phase at cooling. This phase transition is a second-order phase transition.

In all known smectics C the angle between the director and the normal to the layers is small. In other words, the C phase is always close to the A phase. This means that when studying the C phase it is necessary to take into account critical effects, associated with the A–C transition. These critical effects manifest themselves particularly strongly in dynamics.

Genuine smectics, such as smectics A and C, are phases where there is one-dimensional density modulation. As was shown by Peierls and Landau (see Landau and Lifshitz 1980), long-wavelength fluctuations in such a system destroy the genuine long-range order. Nevertheless, the elasticity with respect to the smectic layer displacement is well defined. As was demonstrated by Grinstein and Pelcovits (1982) and one of the authors (EK) (Kats 1982), the appropriate elasticity moduli depend only weakly (logarithmically) on scale.

Long-wavelength fluctuations of smectic layers manifest themselves particularly strongly in the dynamics of smectics. As was first shown by Mazenko *et al* (1983), these

fluctuations bring about emergence of $\omega^{-1}$ diverging corrections to the so-called bulk viscosity coefficients of smectics. This leads to anomalous frequency dependence of the attenuation of both first and second sound. The self-consistent theory of dynamical fluctuation effects in smectics, accounting for the logarithmic renormalization of the parameters, was developed by two of the authors (EK and VL) (Kats and Lebedev 1983, 1988).

Strictly speaking these papers describe the smectic A phase. Therefore there is a problem of peculiarities of the anomalous attenuation of sound in the smectic B and C phases, which in comparison with the A phase have an additional long-wavelength orientational mode. This problem was investigated by two of us (EK and VL) in Kats and Lebedev (1985) where we have come to the conclusion that the expressions describing the anomalous attenuation of sound in the B and C phases practically coincide with the expressions for the A phase. This conclusion is valid for hexatic smectics B but requires a more thorough analysis for the C phase.

However, while studying the spectrum of the B and C phases we have ignored the coupling of the orientational and smectic (undulation) modes. This coupling is, of course, missing in hexatics. The weakness of this coupling in the C phase has been accounted for in Kats and Lebedev (1985) by the small value of the tilt angle of the director in real smectics C. However, the study of the critical dynamics of the smectic A-to-smectic C transition, performed by us in Gurovich *et al* (1988), has shown that, despite the small value of the tilt angle of the director, the coupling of the orientational and undulational modes in the C phase is far from being weak. This brought us back to the problem of peculiarities of the anomalous attenuation of sound in the smectic C phase.

The first experiments in which the $\omega^{-1}$ law was observed (Gallani and Martinoty 1985, Bhattacharya and Ketterson 1982) concerned the A phase. Yet, recently Collin *et al* (1986a) have observed the anomalous attenuation of sound in the C phase. It is important that they have been the first to observe previously the anomalous attenuation of sound in the A phase of the same substance (Collin *et al* 1986b), which makes it possible to compare the character of this attenuation in the A and C phases.

In this paper we shall carry out a consistent calculation of the anomalous attenuation of sound in the C phase. In this calculation we shall employ the non-linear dynamic equations of the C phase on the basis of the diagram technique. The results hold both for the spectrum of the first and second sound and for the spectrum of relaxation (orientational and undulation) modes. Comparison with experimental data will be given in the conclusion.

## 2. Thermodynamics of smectic C

It is convenient to describe the layered structure of smectics by means of the density modulation phase; this variable will be represented by $W$. By virtue of the definition of $W$ the condition $W(t, r) = $ constant sets a position in space and evolution in time of a certain smectic layer. Accordingly the vector $\nabla W$ fixes the direction of the normal to the layer, and

$$l = \frac{\nabla W}{|\nabla W|} \tag{1}$$

is a unit vector along this direction.

The anisotropy of a liquid crystal is characterized by the director, pointing to the preferred direction of the long axes of molecules. In smectics A the director is orthogonal to the layer, i.e. coincides with the vector $l$. In smectics C the director $n$ is tilted by a certain angle to the normal, therefore the vector

$$\psi = n \times l \tag{2}$$

is non-zero. The modulus of the vector $\psi$ determines the tilt angle of the director, and the unit vector

$$n_1 = \frac{\psi}{|\psi|} \tag{3}$$

fixes the preferred direction in the plane of the smectic layer.

In the C phase the modulus $\psi$ is fixed but the direction of the vector $n_1$ is not fixed. Thus the macroscopic state of smectic C should be characterized by an additional variable in comparison with the A phase, characterizing the direction of $n_1$. This is accounted for by spontaneous breaking of the rotational symmetry of the layer in smectic C.

By virtue of the conditions $n_1^2 = 1$, $n_1 \cdot l = 0$, the vector $n_1$ has only one degree of freedom which we shall call orientational. This degree of freedom can be conveniently described by means of the non-holonomic angle $\varphi$ whose variation is set as follows:

$$\delta\varphi = 2(n_1 \times l)\,\delta n_1. \tag{4}$$

The consequence of definition (4) is the variation commutation rule:

$$(\delta_1\delta_2 - \delta_2\delta_1)\varphi = \frac{2}{|\nabla W|^2}\,\varepsilon_{ikn}l_i\delta_1\nabla_k W\delta_2\nabla_n W. \tag{5}$$

Apart from the mentioned variables $W$, $\varphi$ the overall set of long-wavelength variables of smectic includes the momentum density $j$, the mass density $\rho$ and specific entropy $\sigma$. Thus the energy of the C phase is written down as

$$H = \int d^3r \left(\frac{j^2}{2\rho} + E(\rho, \sigma, \nabla W, \nabla_i\nabla_k W, n_1\nabla\varphi)\right). \tag{6}$$

The energy density $E$ depends only on derivatives of $W$ since the energy $E$ does not change at the transformation $W \rightarrow W +$ constant. The gradient of the vector $n_1$ is expressed via gradients of $\varphi$ and $W$.

In our approximation the energy density $E$ can be conveniently represented as a sum:

$$E = \varepsilon(\rho, \sigma) + E_W + E_\varphi + E_{\varphi W} \tag{7}$$

of the isotropic term $\varepsilon$, smectic energy $E_W$, orientational energy $E_\varphi$ and of the cross term $E_{\varphi W}$.

The smectic part of the energy of the C phase in our approximation can be written as follows:

$$E_W = (B/8)[q_s^{-2}(\nabla W)^2 - 1]^2 + (1/2q_s^2)K\delta_{ik}^{\perp}\delta_{nm}^{\perp}\nabla_i\nabla_k W\nabla_n\nabla_m W. \tag{8}$$

Here $B$ is the modulus of smectic layer compression, $K$ is the elasticity modulus, $q_s$ is

the wavevector, determining the modulation period of the smectic; this period being equal to $2\pi q_s^{-1}$. In (8) and henceforth we use the following designation:

$$\delta_{ik}^{\perp} = \delta_{ik} - l_i l_k.$$

The orientation energy $E_\varphi$ can be written down as follows:

$$E_\varphi = \tfrac{1}{2}[(\alpha_1 - \alpha_2)n_{1i}n_{1k} + \alpha_2 \delta_{ik}^{\perp} + \alpha_3 l_i l_k]\nabla_i \varphi \nabla_k \varphi. \tag{9}$$

The quantities $\alpha_{1,2,3}$ involved here can be called orientational elasticity moduli. The cross term $E_{\varphi W}$ in (7) is represented as

$$E_{\varphi W} = -\beta q_s^{-1} \boldsymbol{n}_1 \nabla \varphi \delta_{ik}^{\perp} \nabla_i \nabla_k W. \tag{10}$$

Equation (10) is written down in the main (isotropic) approximation. The coefficient $\beta$ is the cross-elasticity modulus. Anisotropic coefficients of the cross elasticity are small as long as the smectic C is close to the point of the second-order phase transition into the A phase (see section 4).

The energy minimum (8) is achieved by the solution $W = q_s z$, which describes the system of equidistant smectic layers, perpendicular to the $z$ axis. To describe deviations of smectic layers from this position one should put

$$W = q_s(z - u). \tag{11}$$

Here $u$ is a displacement vector of layers along the $z$ axis.

Expanding the energy (8) in $u$, we get in the second order the following expression:

$$E_W^{(2)} = (B/2)(\nabla_z u)^2 + (K/2)[(\nabla_x^2 + \nabla_y^2)u]^2. \tag{12}$$

In the same quadratic approximation equations (9) and (10) reduce to

$$E_\varphi^{(2)} = \tfrac{1}{2}\alpha_2(\nabla_x \varphi)^2 + \tfrac{1}{2}\alpha_1(\nabla_y \varphi)^2 + \tfrac{1}{2}\alpha_3(\nabla_z \varphi)^2$$
$$E_{\varphi W}^{(2)} = \beta \nabla_y \varphi(\nabla_x^2 + \nabla_y^2)u. \tag{13}$$

As has been shown in Grinstein and Pelcovits (1982) and Kats (1982) on the basis of perturbation theory, in smectic A there is logarithmic renormalization of the moduli $B$, $K$. This effect is due to thermal fluctuations of $u$ and is associated with the third- and fourth-order interaction terms which emerge from the expansion of the first term in the energy (8) in $u$. A similar situation takes place in a smectic C, but the renormalization group equations become complicated because of coupling of the smectic and orientational degrees of freedom. The analysis of the renormalization group equations obtained will be published elsewhere.

Bearing in mind the real situation, from here on we shall regard this logarithmic renormalization as weak.

## 3. Dynamic equations

Now we can take up derivation of the long-wavelength equations of dynamics for smectics C. Then we shall largely follow the work of Kats and Lebedev (1985) retaining, however, a number of the omitted terms. First we shall derive non-dissipative (reactive) terms in the equations of the dynamics and then add the dissipative terms to them. Formally these terms differ from each other by symmetry with respect to time reversal.

It is most convenient to derive the non-dissipative terms in the equations by the Poisson brackets method (see the review by Dzyaloshinskii and Volovik 1980). By means

of this method one can derive local equations automatically satisfying all necessary conservation laws. For the variables $j$, $\rho$, $\sigma$, $W$ describing smectics the non-zero brackets are:

$$\{j_i(r_1), \rho(r_2)\} = -\nabla_i \rho \delta(r_1 - r_2) + \nabla_i \delta(r_1 - r_2)\rho(r_2)$$

$$\{j_i(r_1), \sigma(r_2)\} = -\nabla_i \sigma \delta(r_1 - r_2)$$

$$\{j_i(r_1), w(r_2)\} = -\nabla_i W \delta(r_1 - r_2) \tag{14}$$

$$\{j_i(r_1), j_k(r_2)\} = -\nabla_i j_k \delta(r_1 - r_2) + \nabla_i \delta(r_1 - r_2)j_k(r_2)$$
$$\qquad + \nabla_k \delta(r_1 - r_2)j_i(r_2).$$

Note the local and universal character of the expressions given in (14).

The non-dissipative equations are invariant with respect to time reversal. Therefore the expression for the Poisson bracket must change its sign at $t \to -t$. The Poisson bracket (14) obeys this condition. It follows from this condition that to derive the non-dissipative equations for smectic C one should add to (14) the only non-zero bracket $\{j, \varphi\}$ whose expression reads

$$\{j_i(r_1), \varphi(r_2)\} = -\nabla_i \varphi \delta(r_1 - r_2) + \nabla_k \delta(r_1 - r_2)R_{ik}(r_2) \tag{15}$$

where

$$R_{ik} = \varepsilon_{lki} l_l + \lambda_1 (n_{1i} l_k + n_{1k} l_i) + \lambda_2 [n_{1i}(l \times n_1)_k + n_{1k}(l \times n_1)_i]. \tag{16}$$

Here $\lambda_1$, $\lambda_2$ are phenomenological parameters which are functions of $\rho$, $\sigma$.

Now we can write in full the non-dissipative equations for smectic C which are the Liouville equations with the Hamiltonian (6). They have the form

$$\partial \rho / \partial t = \{H, \rho\} = -\nabla_i (\rho V_i) \tag{17}$$

$$\partial \sigma / \partial t = \{H, \sigma\} = -V_i \nabla_i \sigma \tag{18}$$

$$\partial W / \partial t = \{H, W\} = -V_i \nabla_i W \tag{19}$$

$$\partial \varphi / \partial t = \{H, \varphi\} = -V_i \nabla_i \varphi - R_{ik} \nabla_k V_i \tag{20}$$

$$\partial j_i / \partial t = \{H, j_i\} = -\nabla_k (T_{ik} - \nabla_j S_{jik}). \tag{21}$$

Here $V = j/\rho$ is the velocity and the tensors $T_{ik}$, $S_{jik}$ are:

$$T_{ik} = P\delta_{ik} + \rho V_i V_k + \frac{\partial E}{\partial \nabla_k W} \nabla_i W + 2 \frac{\partial E}{\partial \nabla_k \nabla_j W} \nabla_j \nabla_i W$$

$$+ \frac{\partial E}{\partial \nabla_k \varphi} \nabla_i \varphi + \frac{\partial E}{\partial \nabla_j \varphi} \nabla_j \varphi l_i l_k + \varepsilon_{kij} \frac{\partial E}{\partial \nabla_m \varphi} \frac{\nabla_m \nabla_j W}{|\nabla W|}$$

$$+ \varepsilon_{ikm} \frac{\partial E}{\partial \nabla_j \varphi} \frac{\nabla_n \nabla_j W}{|\nabla W|} l_m l_n + \varepsilon_{nkm} \frac{\partial E}{\partial \nabla_j \varphi} \frac{\nabla_n \nabla_j W}{|\nabla W|} l_m l_i \tag{22}$$

$$+ \nabla_j \frac{\partial E}{\partial \nabla_j \varphi} \{\lambda_1 (n_{1i} l_k + l_i n_{1k}) + \lambda_2 [n_{1i}(l \times n_1)_k + n_{1k}(l \times n_1)_i]\}$$

$$S_{jik} = \frac{\partial E}{\partial \nabla_k \nabla_j W} \nabla_i W + \varepsilon_{ikm} \frac{\partial E}{\partial \nabla_j \varphi} l_m.$$

The pressure $P$ entering in (22) is standardly defined as

$$P = \rho \frac{\partial E}{\partial \rho} - E. \tag{23}$$

It is easily seen that $\delta\theta_i \varepsilon_{ikn} T_{kn}$ determines the variation of the energy density $E$ at the rotation by an angle $\delta\theta_i$. By virtue of the rotational invariance of $E$ it means that this combination is zero, i.e. the tensor $T_{ik}$ is symmetric. The term $\nabla_k \nabla_j S_{jik}$ in the right-hand side of (21) can always be reduced to the divergence of the symmetric tensor. Thus $\partial j_i/\partial t$ cn be written as the divergence of the symmetric tensor, which guarantees fulfilment of the angular momentum conservation law.

To the non-dissipative equations one should add the kinetic terms. With these terms taken into account the equations become

$$\frac{\partial \varphi_a}{\partial t} = \{H, \varphi_a\} - \Gamma_{ab} h_b. \tag{24}$$

In equation (24) $\varphi$ implies the overall set of long-wavelength variables (for smectic C they are $j$, $\rho$, $\sigma$, $W$, $\varphi$), the summation is performed over the repeated index $b$. The molecular field $h_a$, conjugated to $\varphi_a$ in (24), equals

$$h_a = \delta H/\delta \varphi_a. \tag{25}$$

The differential operator $\Gamma_{ab}$ in (24) is defined by the set of kinetic coefficients (of viscosity, thermal conductivity, permeation; see Kats and Lebedev 1983, 1988).

For the orientational variable $\varphi$ and the momentum density $j$ the dynamic equations with the kinetic terms taken into account acquire the following form:

$$\partial \varphi/\partial t = V_i \nabla_i \varphi - R_{ik} \nabla_i V_k - \Gamma h_\varphi \tag{26}$$

$$\partial j_i/\partial t = -\nabla_k (T_{ik} - \nabla_n S_{ink}) + \nabla_k (\eta_{iklm} \nabla_m V_l). \tag{27}$$

In (26) there is a kinetic coefficient $\Gamma$; the quantity $\Gamma^{-1}$ has the dimensionality of viscosity and is analogous to the torsional viscosity of nematics. The viscosity tensor in the right-hand side of equation (27) has the following structure (using the designations of Forster *et al* 1971):

$$\eta_{iklm} = \eta_1 l_i l_k l_l l_m + \eta_2 (\delta_{il}^{\perp} \delta_{km}^{\perp} + \delta_{kl}^{\perp} \delta_{im}^{\perp})$$
$$+ \eta_3 (l_i l_l \delta_{km}^{\perp} + l_k l_l \delta_{im}^{\perp} + l_i l_m \delta_{kl}^{\perp} + l_k l_m \delta_{il}^{\perp})$$
$$+ (\eta_4 - \eta_3) \delta_{ik}^{\perp} \delta_{lm}^{\perp} + 2\eta_5 (\delta_{lm}^{\perp} l_i l_k + \delta_{ik}^{\perp} l_l l_m). \tag{28}$$

Generally speaking, by virtue of the biaxiality of smectic C the structure of the viscosity tensor of this phase is more complicated. Yet, due to the small value of the tilt angle of the director with respect to $l$, we shall confine ourselves to the uniaxial expression (28).

## 4. Critical behaviour

In this section we shall briefly describe the behaviour of the parameters of the smectic C phase near $T_c$ (the temperature of transition into the A phase). Details can be found

in our works (Gurovich *et al* 1988; Kats and Lebedev 1986) devoted to theoretical investigation of the A–C transition. Note that the conclusions resulting from the consideration of this transition and concerning the C phase are largely general; due to the small value of the tilt angle of the director in real smectics C they can always be regarded as close to smectics A.

The order parameter, describing the transition from smectic A to smectic C, is the vector $\psi$ introduced into (2). The mean value of $\psi$ is zero in the A phase and is non-zero in the C phase. By virtue of the definition $\langle\psi\rangle$ sets an average tilt of the director $n$ with respect to the normal $l$ to smectic layers. It is not difficult to see that in the expansion of the free energy there are only $\psi$ even terms. Therefore the A–C transition is a second-order transition.

Due to the condition $l\psi = 0$ the order parameter $\psi$ has two components. Yet the critical behaviour of smectics at the A–C transition is not described by the standard two-component model $\psi^4$—the order parameter $\psi$ is defined in the real but not isotopic space, therefore it 'anchors' with vector quantities.

Non-universality of the behaviour of smectic characteristics at A–C transition reveals itself both in the mean field theory and in a wide region of developed critical fluctuations where corrections to the gradient terms can be discarded. In this region the critical behaviour of the parameters is described by the non-universal indices (which are functions of $\alpha_1/\alpha_2$). The universal behaviour corresponding to the $\psi^4$ model with a two-component-order parameter takes place only in a very narrow region near $T_c$ where corrections to gradient terms become relevant. The region of universal behaviour is apparently not achieved experimentally, therefore we shall not consider it. By the region of developed fluctuations we mean the region of the non-universal critical behaviour.

In the mean field theory in the C phase, $\langle\psi\rangle^2 \sim |T_c - T|$. In the region of developed fluctuations:

$$\langle\psi\rangle \sim |T_c - T|^\beta. \tag{29}$$

The value of the exponent $\beta$ is 0.43–0.5. In (8) the wavevector $q_s$ has been introduced. The critical correction to it is determined by the law:

$$\Delta q_s \sim \langle\psi\rangle^2. \tag{30}$$

In the mean field theory $\Delta q_s \sim |T_c - T|$; in the region of developed fluctuations, $\Delta q_s \sim |T_c - T|^{2\beta}$.

Quantities like heat capacity or compressibility of smectics, in the mean field theory, experience a positive jump at the transition into the C phase. The modulus $B$ decreases at this transition by a value of the order of itself. In the region of developed fluctuations there arise critical singularities in the heat capacity $C$ or in the quantity $B^{-1}$:

$$C, B^{-1} \sim |T_c - T|^\alpha. \tag{31}$$

These singularities reveal themselves both in the A phase and in the C phase but with different factors at $|T_c - T|^\alpha$. The exponent of the heat capacity $\alpha$ is small ($\alpha < 0, 1$). Therefore experimental observation of the singularity (31) is difficult.

The elasticity moduli $K$, $\alpha$ and $\beta$ entering in (8), (9) and (10) can be expressed via Frank constants $K_{1,2,3}$ and the order parameter $\psi$ of the A–C transition. This representation is useful because there are no corrections to the Frank coefficients $K_{1,2,3}$ in the mean field theory and they are negligibly small in the region of developed fluctuations (Kats and Lebedev 1986).

Substituting into the Frank energy

$$\tfrac{1}{2}\{K_1(\nabla n)^2 + K_2[n(\nabla n)]^2 + K_3[n(\nabla n)]^2\}$$

the director $n$ as a function of the smectic variable $u$ and of the angle $\varphi$ in accordance with the definitions (1)–(4) and (11), we get

$$K = K_1 \qquad \alpha_{1,2,3} = \tfrac{1}{4}K_{2,1,3}\langle\psi\rangle^2 \qquad \beta = \tfrac{1}{2}K_1\langle\psi\rangle. \tag{32}$$

Note that these expressions are valid only in the intermediate asymptotic region mentioned above.

Quite different is the critical behaviour of the viscosity coefficients. There are no critical singularities in the coefficients $\eta_2$, $\eta_3$, determining the dispersion law of transverse to the wavevector components of the velocity. The coefficients $\eta_1$, $\eta_4$, $\eta_5$ determining the attenuation of the first and second sound have a strongly pronounced critical behaviour. In the mean field theory in the C phase (but not in the A phase!) corrections emerge to these coefficients:

$$\Delta\eta_1, \Delta\eta_4, \Delta\eta_5 \sim |T - T_c|^{-1}.$$

In the region of developed fluctuations:

$$\Delta\eta_1, \Delta\eta_4, \Delta\eta_5 \sim |T_c - T|^{-\alpha - z\nu}. \tag{33}$$

Here $z$ is the dynamical exponent, close to two, $\nu$ is the exponent of the critical radius. In the mean field theory $\nu = \tfrac{1}{2}$, in the region of developed fluctuations $\nu = 0, 62$–$0, 64$. The fluctuational divergence (33) occurs both in the A phase and in the C phase (but with different factors).

The critical behaviour of the kinetic coefficient $\Gamma$, entering in equation (26) for the angle $\varphi$ is

$$\Gamma = 4(\gamma_1\langle\psi\rangle^2)^{-1}. \tag{34}$$

Here $\gamma_1$ is the (nematic) coefficient of torsion viscosity estimated as $\gamma_1 \sim \eta_{2,3}$. For the non-dissipative parameters $\lambda_1$, $\lambda_2$ entering in (16), (22) there is the following expression (Gurovich *et al* 1988):

$$\lambda_1 = \frac{\lambda + 1}{\langle\psi\rangle} \qquad \lambda_2 = \lambda. \tag{35}$$

Here $\lambda$ is the nematic non-dissipative parameter close to unity for substances consisting of rod-like molecules. Let us stress that the quantities $\gamma_1$, $\lambda$ in (34), (35) have no critical singularities.

The estimates given are characteristic of the low-frequency (hydrodynamic) region. If the characteristic frequencies $\omega$ exceed the inverse time of the order parameter relaxation, the critical corrections to the parameters become frequency-dependent. We shall not give the appropriate estimates, noting only that the frequency suppresses the mean field and fluctuational contributions.

## 5. Spectrum of slow modes

The presence of the long-wavelength orientational degree of freedom in smectic C causes the appearance of an extra, low-frequency mode in comparison with the A phase. This

mode has a diffusive character as in nematics, and it is slow, which is accounted for by the small value of the parameter

$$K\rho/\eta^2 \ll 1. \tag{36}$$

Here $\rho$ is the density, $K$ is a quantity of the order of the Frank modulus and $\eta$ is the characteristic viscosity.

In smectics the second sound also propagates. For propagation angles close to 0° or 90° with respect to smectic layers, this acoustic mode transforms into two diffusion modes. One of these modes is fast and is associated with shear velocity relaxation. The other mode is slow and is associated with the smectic layer relaxation. For the propagation direction along the layers this mode is undulational. It is with this mode that dynamical fluctuational effects in smectics are associated.

Below we shall be interested in characteristics of the slow undulational and orientational modes in smectic C. These are low-frequency modes, therefore we can consider that

$$P = \text{const.} \qquad T = \text{const.} \qquad \nabla j = 0. \tag{37}$$

Thus, to describe these modes it is necessary to use equations (19), (26), (27) (the terms associated with permeation emerging in the right-hand side of equation (19) are not relevant for our study). We shall need only the vortex component of equation (27); to obtain that we must multiply this equation by the transverse projection operator $\delta_{ij} - \nabla_i\nabla_j/\nabla^2$.

The spectrum of eigenmodes of the system is determined by linearized equations of dynamics. We shall choose as the variables describing the linear dynamics of the slow mode, components of the velocity $V_z$, $V_t$, the displacement vector $u$ introduced by equation (11) and the angle $\varphi$, introduced by (4). The non-holonomity of the latter variable in the given case is irrelevant; in the linear approximation, the angle $\varphi$ itself can be used as an independent variable. Note that the $z$ axis at equilibrium is directed along the normal to the smectic layers, i.e. $l$ is a unit vector along this axis. The component $V_t$ is defined in the Fourier representation as

$$V_t = (l \times q)V/q_\perp. \tag{38}$$

Here $q$ is the wavevector, $q_\perp^2 = q_x^2 + q_y^2$.

As follows from equation (35), in real smectics C where the tilt angle of the director is small, $\lambda_1 \gg \lambda_2$. Therefore linearizing equation (26) for the angle $\varphi$ we omit the term with $\lambda_2$. Due to the inequality $\lambda_1 \gg 1$ we may omit the term generated by the first term in the right-hand side of (16). Assuming that at equilibrium the $y$ axis is directed along $n_1$, we find the following equation for the angle $\varphi$ in the Fourier representation:

$$\omega\varphi = \lambda_1[(q_xq_z/q_\perp)V_t + q_yV_z] + i\Gamma h_\varphi. \tag{39}$$

Here we have taken into account the condition $\nabla V = 0$ and in conformity with (25) introduced the molecular field $h_\varphi$:

$$h_\varphi = (\alpha_2 q_x^2 + \alpha_1 q_y^2 + \alpha_3 q_z^2)\varphi + i(\beta/2)q_yq_\perp u. \tag{40}$$

The molecular field conjugate to the smectic variable $U$ is

$$h_U = (Bq_z^2 + Kq_\perp^4)u - \tfrac{1}{2}i\beta q_yq_\perp^2 \varphi. \tag{41}$$

Linearization of equation (19) in the Fourier representation yields

$$Wu = iV_z. \tag{42}$$

To equations (39), (42) one should add equations for the components $V_t$, $V_z$ arising from

(27). Studying the slow modes one can omit the term with the time derivative and should bear in mind also the incompressibility condition $\nabla V = 0$. Employing equations (8), (9) and (10) to construct the non-dissipative stress tensor entering in (27) and linearizing it over $\varphi$, $u$, we ultimately get the following:

$$i\lambda_1(q_z q_x/q_y)h_\varphi = (\eta_2 q_\perp^2 + \eta_3 q_z^2)V_t$$
$$h_U + i\lambda_1 q_y h_\varphi = (\eta_2 q_\perp^2 + \eta_3 q_z^2)V_z. \tag{43}$$

From (39), (42) and (43) one can find the spectrum of slow modes for smectic C. Strictly speaking, equations (42) and (43) hold only for the region of existence of the undulational mode, i.e. at

$$q_z \ll \eta_3 q_\perp^2/B\rho. \tag{44}$$

Under this condition $q_z \ll q_\perp$; that is why it follows from (42) that $V_t = 0$ and in (43) one can ignore the term $\eta_1 q_z^2$.

The resultant system describes the two branches of the spectrum, corresponding to the undulational and orientational modes. The dispersion laws for these branches read:

$$\omega_\pm(q) = \tfrac{1}{2}(\omega_1 + \omega_2) \pm \tfrac{1}{2}\left((\omega_1 - \omega_2)^2 + \frac{q_y^2}{q_\perp^2}\frac{4m\omega_1\omega_2}{1+m}\right)^{1/2}. \tag{45}$$

Here the designations

$$m = [(\lambda_1/2) + \beta(q_y^2/\hat\alpha q^2)]^2/\Gamma\eta_3 \tag{46}$$

$$\omega_1 = -i\frac{Bq_z^2 + \tilde K q_\perp^4}{\eta_3 q_\perp^2}$$

$$\omega_2 = -i\Gamma[1 + m(q_y^2/q_\perp^2)]\hat\alpha q^2 \tag{47}$$

$$\tilde K = K - \beta^2 q_y^2/\hat\alpha q^2$$

are introduced. By virtue of the inequality $q_z \ll q_\perp$ in (46) and (47) one must insert $\hat\alpha q^2 = \alpha_1 q_x^2 + \alpha_2 q_y^2$.

According to the definition (46) with the explicit equations (32) and (34) for $\Gamma$, $\beta$ and $\lambda_1$ taken into account we have

$$m = \left(\frac{\lambda+1}{2} + \frac{K_1 q_\perp^2}{K_1 q_y^2 + K_2 q_x^2}\right)^2 \frac{\lambda_1}{4\eta_3}.$$

Remembering that $\eta_3 \sim \gamma_1$, $\lambda \sim 1$ we conclude that $m \sim 1$. From equations (32) and (34), and from the estimates $\eta_3 \sim \gamma_1$ and $m \sim 1$, at $Bq_z^2 \sim Kq_\perp^4$ the frequencies $\omega_1$, $\omega_2$ are of the same order and do not depend on the proximity to the point of transition into the A phase.

In the region of the wavevectors $q_z \gg \eta_3 q_\perp^2/B\rho$ instead of the undulational mode there is a propagating second sound. In this case the orientational mode is described by the dispersion law

$$\omega = -i\Gamma\hat\alpha q^2 \tag{48}$$

where $\hat\alpha q^2 = \alpha_1 q_x^2 + \alpha_2 q_y^2 + \alpha_3 q_z^2$. Since the region (44) is very narrow, it is the dispersion law (48) that must be observed in optics. Light-scattering experiments in smectics C (Chandrasekhar 1977) at least qualitatively confirm the dispersion law (48) and the estimates given above.

As follows from (45) the coupling of the undulational and orientational modes is strong. Yet at $Bq_z^2 \gg Kq_\perp^4$ (which, as can be easily seen, is compatible with (44)) this coupling vanishes and the dispersion law $\omega_-(q)$ coincides with (48).

## 6. Diagram technique

The dynamic effects associated with fluctuations of smectic layers can be conveniently studied by means of the diagram technique. The diagram technique, suitable for such problems, was first constructed by Wyld (1961). We shall employ the formalism proposed by Lebedev *et al* (1983). Having the non-linear dynamic equations (24) for smectic C, we can construct the effective Lagrangian

$$L = p_a \left( \frac{\partial \varphi_a}{\partial t} - \{H, \varphi_a\} + \Gamma_{ab} h_b \right) + iTp_a \Gamma_{ab} p_b. \tag{49}$$

Here $\varphi_a$ is an overall set of long-wavelength variables for smectic C and $p_a$ are the additional fields conjugated to them.

By means of the Lagrangian (49) one can calculate the dynamic correlations functions of the long-wavelength variables. So for the pair correlation functions there are the following expressions:

$$D_{ab}(t_1 - t_2, r_1 - r_2) = \langle \varphi_a(t_1, r_1) \varphi_b(t_2, r_2) \rangle = \int D\varphi \, Dp \, \exp\left( i \int dt \, d^3r \, L \right) \varphi_a \varphi_b$$

$$\tag{50}$$

$$G_{ab}(t_1 - t_2, r_1 - r_2) = \langle \varphi_a(t_1, r_1) p_b(t_2, r_2) \rangle = \int D\varphi \, Dp \, \exp\left( i \int dt \, d^3r \, L \right) \varphi_a \varphi_b.$$

The function $D_{ab}$ is a pair correlation function of the long-wavelength variables, $G_{ab}$ determines the linear susceptibility of the system. Consequently, the poles $G_{ab}(\omega, k)$ fix the spectrum of eigenmodes of the system. Note that the correlation function $\langle p_a p_b \rangle$ is identically equal to zero (Khalatnikov *et al* 1984).

The definition (50) enables us to formulate the standard diagram technique for calculating the correlation functions. This technique involves bare correlation functions determined by the part of the Lagrangian (49), quadratic in the fields $\varphi_a$, $p_a$, and the interaction vertices are determined by the high-order terms of the expansion of (49) in these fields. We are interested in the role of the corrections to the long-wavelength dynamics of smectic C, determined by this diagram.

The analysis shows that the important fluctuational effects are associated only with fluctuations of smectic layers. This means that calculating the correlation functions in conformity with (50) we must retain in the Lagrangian the anharmonic terms over the smectic variable $W$ and confine ourselves to the quadratic approximation over the other (weakly fluctuating) variables. Therefore integration over these weakly fluctuating variables in (50) becomes Gaussian and can be performed explicitly. This procedure of effectively excluding the weakly fluctuating variables was proposed by Kats and Lebedev (1983, 1988).

Later in this section we shall be interested in the correlation functions of the variables $W$, $\varphi$ and of the fields conjugate to them. These correlators describe the slow modes of smectic C in the region of wavevectors defined by the inequality (44); it is in this region that the main fluctuation corrections are obtained. In this case the procedure of excluding

weakly fluctuating variables is analogous to the derivation of linear dynamic equations for slow modes, performed in the preceding section.

Due to the Gaussian character of integration over the weakly fluctuating variables, the procedure of calculating integrals over them in (50) reduces to the replacement of the Lagrangian $L$ by its extremum over these fields. Calculation of the extremum of the Lagrangian (49) over the fields $p_\rho$, $p_\sigma$, $p_l$ ($p_l$ is conjugated to $j_l$, vortex free part of $j$) brings about the conditions (37). Then the condition of the extremum (49) over $\rho$, $\sigma$ and $\nabla j$ reduces to $p_\sigma = p_\rho = p_l = 0$. Exclusion of the velocity components $V_z$, $V_t$ and of the fields conjugate to them should be performed with the explicit form of the viscosity tensor (28) taken into account.

The result of calculating the extremum of the Lagrangian (49) over these fields can be represented as

$$L_{\text{eff}} = p_\mu \gamma_{\mu\nu}(\partial \varphi_\nu/\partial t) + p_\mu(\delta E/\delta \varphi_\mu) + iT p_\mu \gamma_{\mu\nu} p_\nu. \qquad (51)$$

Here the subscripts $\mu$, $\nu = 1, 2$ and $(\varphi_1, \varphi_2) = (u, \varphi)$ while $p_1, p_2$ are supplementary fields (not coinciding with the fields $p_u$, $p_\varphi$ conjugate to $u$, $\varphi$). In (51) there are variational derivatives of the energy $E$ over $u$ and $\varphi$; explicit expressions for these derivatives can be found by means of (9), (10). The matrix $\gamma_{\mu\nu}$ in (51) is the following differential operator:

$$\gamma_{\mu\nu} = \begin{pmatrix} -\eta_3 \nabla_\perp^2 - \lambda_1^2 \Gamma^{-1} \nabla_y^2 & \Gamma^{-1} \lambda_1 \nabla_y^2 \\ -\Gamma^{-1} \lambda_1 \nabla_y^2 & \Gamma^{-1} \end{pmatrix}. \qquad (52)$$

In the smectic B phase there is no coupling of the undulational and orientational modes, which corresponds to $\lambda_1 = 0$. In this case the Lagrangian (51) is the sum of two terms $L_W + L_\varphi$, corresponding to the undulational and orientational modes, respectively. The expression for $L_W$ has the same form in smectics B as in smectics A. Hence it follows that the dynamic effects associated with fluctuations of smectic layers are described by the same expressions in the B phase as in the A phase.

Correlation functions of the fields $u$, $\varphi$, $p_1$, $p_2$ can be calculated by means of (51) in accordance with the definition analogous to (50). Further we shall need the following correlation functions:

$$G(t_1 - t_2, r_1 - r_2) = \langle u(t_1, r_1) p_1(t_2, r_2) \rangle$$

$$= \int Du \, D\varphi \, Dp_1 Dp_2 \exp\left(i \int dt \, d^3r \, L_{\text{eff}}\right) u p_1$$

$$D(t_1 - t_2, r_1 - r_2) = \langle u(t_1, r_1) u(t_2, r_2) \rangle \qquad (53)$$

$$= \int Du \, D\varphi \, Dp_1 Dp_2 \exp\left(i \int dt \, d^3r \, L_{\text{eff}}\right) uu.$$

$$(54)$$

As has been pointed out, the poles of the response function $G(\omega, q)$ determine the spectrum of the system and, in the given case, the spectrum of slow modes.

The Lagrangian (51) describes purely relaxational dynamics of the fields $u$, $\varphi$ (then the kinetic coefficients depend quite non-trivially on the wavevector). For such systems there is a simple relationship between the response function and pair correlation function:

$$D(\omega) = -(T/\omega)[G(\omega) - G(-\omega)]. \qquad (55)$$

The proof of the relation (55), which in fact is the fluctuation–dissipation theorem, can

be found in our work (Gurovich *et al* 1988). The response function $G(\omega)$ is analytical in the upper semi-plane. Integration of (55) over the frequency $\omega$ with this analyticity taken into account yields the following expression for the one-time correlation function:

$$D(t, t) = - iTG(\omega = 0). \tag{56}$$

## 7. Renormalization of the slow mode spectrum

Due to smectic layer fluctuations there emerge corrections to the dispersion law of the slow mode spectrum (45), obtained in the linear approximation. This effect is associated with self-action of the undulational mode described by the terms, non-linear in the displacement vector, in the dynamic equations. These corrections have a logarithmic character and are, apparently, small in real smectics. Yet the consecutive procedure of taking them into account is no doubt of methodological interest and the expressions for the pair correlation obtained in this section will be further used for calculating the diverging $\omega^{-1}$ corrections to the viscosity coefficients.

Fluctuation corrections to the spectrum can be calculated within the framework of the perturbation theory. The corresponding diagram technique is generated by the Lagrangian (51). In this technique there are third- and fourth-order vertices, defined by (9), (11) and (51). Let us give the explicit form of corresponding contributions to the Lagrangian:

$$
\begin{aligned}
L_{\text{eff}}^{(3)} &= - (B/2)\nabla_z p_1(\nabla u)^2 - B\nabla p_1 \nabla u \nabla_z u \\
L_{\text{eff}}^{(4)} &= (B/2)\nabla p_1 \nabla u (\nabla u)^2.
\end{aligned}
\tag{57}
$$

Remember that the $z$ axis is directed along the (equilibrium) normal to smectic layers.

The bare Green functions are defined by the quadratic part of (51). The explicit equation for the bare correlation function (53) in the Fourier representation reads

$$G = \frac{\omega + i\Gamma(\alpha_1 q_x^2 + \alpha_2 q_y^2 + \alpha_3 q_z^2)}{(\omega - \omega_+)(\omega - \omega_-)\eta_3 q_\perp^2}. \tag{58}$$

The poles of this expression determine bare dispersion laws of slow modes; these equations naturally coincide with (45). The equation for the correlation function (54) can be found by means of (55). In conformity with (56) and (58) the one-time correlation function $D(t, t)$ proves to be equal to

$$D(t, t, \boldsymbol{q}) = T(Bq_z^2 + \check{K}q_\perp^4)^{-1} \tag{59}$$

where $\check{K}$ is defined in (47). This expression can be obtained directly by means of the part of the energy (7), quadratic in $u$.

Due to the interaction terms (57) corrections to the bare correlation function (58) emerge; these corrections can be calculated in perturbation theory. Singling out self-energy blocks from the perturbation series, we come to the conventional relation

$$G^{-1} = G_0^{-1} - \Sigma. \tag{60}$$

Let us stress that in the framework of this technique this relation is not trivial, but is the consequence of the condition $\langle p_a p_b \rangle = 0$. In the lowest order of the perturbation theory

**Figure 1.** The self-energy function.

the contribution to the self-energy function $\Sigma$ is defined by the diagram in figure 1. In this figure the full curve represents the correlation function (53), the broken curve represents the correlation function (54), and the vertices are generated by the third-order term in (57).

The corresponding analytical expression $\Sigma$ can be conveniently divided into two terms:

$$\Sigma_1(\omega, k) = -i \int \frac{d\nu \, d^3q}{(2\pi)^4} \left[ \frac{B^2}{\rho} k_z \left( q^2 - \frac{k^2}{4} \right) G\left( \nu + \frac{\omega}{2}, q + \frac{k}{2} \right) D\left( \nu - \frac{\omega}{2}, q - \frac{k}{2} \right) \right] \qquad (61)$$

$$\Sigma_2(\omega, k) = i \int \frac{d\nu \, d^3q}{(2\pi)^4} \left( \frac{B^2}{\rho} q_z^2 (2k_\perp q_\perp + k_\perp^2)^2 G(\omega + \nu, k + q) D(\nu, q) \right). \qquad (62)$$

It is not difficult to make sure that the integral in (61) is purely logarithmic. Therefore, calculating it (with logarithmic accuracy) one can drop the dependence on the external frequency $\omega$ and on the wavevector $k$ in the integrand expressions. Comparing the obtained expression for $\Sigma_1$ with equation (58) for the bare correlation function we come to the conclusion that in conformity with (60) the presence of $\Sigma_1$ is equivalent to the emergence of the fluctuational corrections to the modulus $B$, equal to $\Sigma_1/k_z^2$.

The situation with the contribution $\Sigma_2$ is somewhat more complex. The expression for $\Sigma_2(0, 0)$ is an integral, diverging at the upper limit. Yet the value of $\Sigma_2(0, 0)$ must be put equal to zero since the presence of this quantity is equivalent to the emergence of the $(\nabla u)^2$ proportional term in the elastic energy (12). But this term is not possible because of the rotational invariance. So, calculating $\Sigma_2(\omega, k)$ one should subtract from (62) the formally divergent constant $\Sigma_2(0, 0)$. Expanding this difference in $\omega, k$, we get in the main approximation

$$\Sigma_2(\omega, k) - \Sigma_2(0, 0) = \Delta K k_\perp^4 + i\Delta\eta_3 \omega k_\perp^2. \qquad (63)$$

This approximation is justified by the fact that the expressions for $\Delta K$ and $\Delta\eta_3$ are determined by purely logarithmic integrals. As follows from (58), (60) and (63) the quantities $\Delta K$ and $\Delta\eta_3$ have the meaning of corrections to the Frank modulus $K$ and the viscosity coefficient $\eta_3$, entering in equation (58) for the bare correlation function.

Calculating the quantities $\Delta B$, $\Delta K$ and $\Delta\eta_3$ one should use the representation (55). Using the analytical properties of the function $G(\nu)$ one can make sure that $\Delta B$ and $\Delta K$ are defined by the same integrals as in the static case. The equation for the quantity $\Delta\eta_3$ does not reduce to those integrals of the one-time correlation functions. This is natural since $\eta$ is a purely dynamic characteristic.

The analysis reveals that the corrections to $K$ and $\eta$ defined by equation (63) are anisotropic in the smectic layer plane. This is what in fact complicates the form of the renormalization group equations for smectic C. We shall not discuss these equations

here, noting only that for real smectics the logarithmic corrections $\Delta K$, $\Delta \eta_3$ are evidently small compared to $K$ and $\eta_3$.

## 8. Fluctuation corrections to the spectrum of sound

Now we shall study fluctuational corrections to the dynamic equations for weakly fluctuating variables. They emerge when interaction modes are taken into account, which is described by the non-linear terms in the equations of motion (24). As was mentioned above the only strongly fluctuating degree of freedom of smectics is associated with smectic layer displacement. This means that it is necessary to take into account only non-linearities over the displacement vector $u$, introduced in (11).

The most important non-linear $u$ term in the dynamic equations is related to the non-dissipative part of the stress tensor (22). The part of this tensor, quadratic in $u$, generates the following third-order contribution in the Lagrangian (49):

$$L^{(3)} = -\tfrac{1}{2}B(\gamma \nabla_i p_i + \nabla_z p_z)(\nabla u)^2 + B\nabla_i p_k(l_i \nabla_k u + l_k \nabla_i u)[\gamma(\delta \rho / \rho) - \nabla_z u] \tag{64}$$

where

$$\gamma = -(\ln q_s / \ln \rho)_\sigma. \tag{65}$$

The derivative in (65) is calculated when the specific entropy $\sigma$ is constant; in this section we shall consider the spectrum of acoustic oscillations at constant $\sigma$.

The acoustic oscillation spectrum is determined by poles of the response function $G_{ik} = \langle j_i p_k \rangle$, where $p_k$ are supplementary fields conjugate to $j_k$. Due to interaction a fluctuational contribution to $G_{ik}$ occurs which can be written in the form of the self-energy function $\Sigma_{ik}$. The latter is defined in such a way that

$$G_{ik}^{-1} = G_{0ik}^{-1} - \Sigma_{ik} \tag{66}$$

where the index zero denotes the bare value. Due to the interaction (64) in the lowest order of the perturbation theory there appears the following contribution to $\Sigma_{ik}$:

$$\Sigma_{ik}(\omega, k) = -(i/\omega)\Sigma_1(\omega, k)[l_i + \gamma(k_i/k_z)][l_k + \gamma(k_k/k_z)]. \tag{67}$$

The function $\Sigma_1(\omega, k)$ is defined by equation (61).

From the preceding section, the function $\Sigma_1$ is related to the fluctuational correction to the modulus $B$. It is easy to verify that the real part of (67) reproduces logarithmic corrections to the modulus $B$ and to the compressibility, which are observed in the static case. In dynamics these quantities determine velocities of the first and second sound. Thus we come to the conclusion that fluctuations cause logarithmic corrections to the velocity of acoustic wave propagation.

The imaginary part of (67) affects the spectrum much more strongly; this imaginary part can be reduced to the following form:

$$\operatorname{Im}\Sigma_{ik}(\omega, k) = -2(\gamma k_i + l_i k_z)(\gamma k_k + l_k k_z)A(\omega, k) \tag{68}$$

where

$$A(\omega, k) = \frac{1}{8T}\int \frac{\mathrm{d}\nu\, \mathrm{d}^3 q}{(2\pi)} B^2 \left(q^2 - \frac{k^2}{2}\right) D\left(\nu + \frac{\omega}{2}, q + \frac{k}{2}\right) D\left(\nu - \frac{\omega}{2}, q - \frac{k}{2}\right). \tag{69}$$

Comparing $\operatorname{Im}\Sigma_{ik}$ in (68) with the structure of the bare correlation function $G_{ik}$ shows

that the presence of (68) is equivalent to the emergence of fluctuational contributions to the following viscosity coefficients:

$$\Delta\eta_1 = 2(1 + \gamma)^2 A \qquad \Delta\eta_4 = 2\gamma^2 A \qquad \Delta\eta_5 = 2\gamma(\gamma + 1)A. \qquad (70)$$

Let us consider the quantity $A(\omega, 0)$, since it is determined by the fluctuational contribution to the absorption of the sound. Using the explicit equations (55) and (58) we find

$$A(\omega, 0) = \frac{1}{|\omega|} \frac{1}{128} \frac{B^{3/2}}{K^{3/2}} \int_0^{2\pi} \left( \frac{1 + [(m_1/m_2) - 1]\cos\theta}{1 + [(m_1 - 1)/(m_2 - 1)]\cos\theta} \right)^{3/2} \frac{\mathrm{d}\theta}{2\pi}. \qquad (71)$$

Here $m_{1,2} = K\alpha_{1,2}/\beta^2$. From (70) and (71) the main conclusion about the dynamics of smectics follows: the divergence $\omega^{-1}$ of the bulk viscosity coefficients due to fluctuations.

Equation (71) holds for the smectic C phase, and the equation for the A phase is obtained from (71) if we put $m_{1,2}^{-1} = 0$. Note that from the explicit form of (71), the coefficient at $B^{3/2}/K^{3/2}|\omega|$ in the expression for $A(\omega, 0)$ for the C phase is always larger than in the A phase.

For information let us give the spectrum of the first and second sound with the fluctuational contribution (70) taken into account:

$$\omega = \pm c_1 k - (\mathrm{i}/\rho k^2)[\tfrac{1}{2}(\eta_2 + \eta_4)k_\perp^4 + (\eta_5 + 2\eta_3)k_z^2 k_\perp^2 + \tfrac{1}{2}\eta_1 k_z^4 + A(\gamma k^2 + k_z^2)^2] \qquad (72)$$

$$\omega = \pm c_2 k - (\mathrm{i}/\rho k^2)[\tfrac{1}{2}\eta_3(k_z^2 - k_\perp^2)^2 + \tfrac{1}{2}(\eta_1 + \eta_2 + \eta_4 - 2\eta_5)k_z^2 k_\perp^2 + Ak_z^2 k_\perp^2]. \qquad (73)$$

Here there are bare viscosity coefficients $\eta$ ($\omega$ independent) and the velocities of the first sound $c_1$, and of the second sound, $c_2$ are defined as

$$c_1^2 = (p/\rho)_\sigma \qquad c_2^2 = \frac{B}{\rho} \frac{k_z^2 k_\perp^2}{k^4}. \qquad (74)$$

Since $A \sim \omega^{-1}$ in the low-frequency limit the fluctuation of the attenuation of sound exceeds the bare attenuation. This means that in (72) and (73) the terms with $\eta$ can be neglected in comparison with the term in $A$.

## 9. Discussion

We were interested in the difference between fluctuational attenuation of sound in smectics A and smectics C. As is known, in the C phase, as compared with the A phase, there is an extra orientational mode whose spectrum is defined by equation (48). However, this fact alone does not affect the expression for the fluctuational attenuation. So, in hexatic smectics B where the orientational mode is also present, the expression for fluctuational attenuation of sound has the same form as in smectics A. Yet in smectics C the situation is more complicated. This fluctuational attenuation is determined by the region of wavevectors in which there is the undulation mode. In smectics C (due to their lower symmetry than in hexatic smectics B) this mode becomes coupled to the orientational mode. This, in particular, brings about distortion of the spectrum (48) for the wavevectors with small $q_z$. The respective region of wavevectors is determined by the inequality (44), while the spectrum in this region is determined by (45). The strong coupling of the orientational and undulational modes takes place despite the weak anisotropy of the layer in real smectics C. This has been demonstrated by our analysis of the critical dynamics of the A–C transition (Gurovich *et al* 1988).

The presence of the strong coupling of the orientational and undulation modes does not alter our conclusion that the bulk viscosity coefficients of smectics diverge as $\omega^{-1}$ due to fluctuations. However this coupling generates the difference of the coefficients at $\omega^{-1}$ in the C phase and in the A phase. The explicit expressions for fluctuational corrections to the viscosity coefficients in smectics C are defined by equations (70) and (71). The integral entering into (71) can only be found numerically, the quantities $m_{1,2}$ being determined by material parameters of each concrete substance.

The fact that the attenuation in smectics C is stronger than in smectics A, follows directly from equation (71). In the single-constant approximation ($\alpha_1 = \alpha_2$), the integral (71) is calculated explicitly:

$$A(\omega, 0) = \frac{T}{\omega} \frac{B^{3/2}}{K^{3/2}} \frac{1}{128} \frac{E(m^{-1})}{1 - m^{-1}}$$

where $E(m^{-1})$ is the complete elliptical first-order integral.

Calculating fluctuational corrections to the viscosities we have confined ourselves to the single-loop approximation. For the calculation to be correct it is necessary that the parameter

$$10^{-2}(TB^{1/2}/K^{3/2}) \tag{75}$$

be small. In real smectics this parameter has the value $10^{-3}$–$10^{-4}$, which is consistent with our approximation. Due to the small value of the parameter (75) the logarithmic corrections to the elasticity modulus and to the viscosity coefficients studied in section 7 are small.

Calculating the integral (69) determining the fluctuational attenuation, we have put $k = 0$. The analysis shows that this is always valid for the study of the spectrum of the first sound. As for the second sound, such neglect is justified only if:

$$k_z/k_\perp \ll \eta_3/\sqrt{K\rho}. \tag{76}$$

In real smectics the parameter in the right-hand side of (76) is of the order $10^2$. Therefore the inequality (76) can be easily satisfied.

Note that only the fluctuational corrections to the viscosity coefficients $\eta_1$, $\eta_4$, $\eta_5$ are large. There is only a small logarithmic correction to the $\eta_3$ viscosity and there are no fluctuational corrections to the viscosity coefficient $\eta_1$ and to the components of the heat conductivity tensor and to the permeation coefficients.

In conclusion let us discuss experimental data of Collin *et al* (1986a, b) for smectic A and C phases of the same substance. Firstly, note that the modulus $B$ in the smectic C is smaller than in the smectic A. This fact is consistent with our analysis of the critical behaviour of the modulus $B$ at the A–C transition (see section 4).

Collin *et al* (1986a, b) concentrated on the analysis of the fluctuational attenuation ($|\omega|^{-1}$) of sound. From the angular dependence of this attenuation they concluded that the dimensionless parameter $\gamma$ entering in (70) has different signs in the A and C phases. In the C phase $\gamma < 0$ and therefore the fluctuational attenuation of the first sound vanishes for certain directions of propagation. This fact is also consistent with our theory. From (65) and (30) it follows that in the mean field theory the coefficient $\gamma$ has a negative jump in the transition point. In the region of developed fluctuations:

$$\gamma \sim |T_c - T|^{2\beta - 1}.$$

Note that the factors in this singularity have different signs in the A and C phases.

In their analysis of the fluctuational attenuation in the smectic C, Collin *et al* (1986a) concluded that the value of the modulus $K$ in the C phase is smaller than its value in the A phase. However our consideration (see section 4) shows that corrections to the modulus $K$ are absent in the single-loop approximation and, therefore, the values of $K$ in A and C phases must be close. This discrepancy disappears if we take into account the fact that in the C phase Collins *et al* used the expression for the fluctuational attenuation which is correct only for the A phase. For the C phase it is necessary to use our equation (71). This equation shows that the factor at

$$(B^{3/2}/K^{3/2})1/\omega$$

for the C phase is always larger than in the A phase. Collin *et al* interpreted this fact as if the modulus $K$ in the C phase is smaller than in the A phase.

Thus our theory shows that in the long-wavelength dynamics of all kinds of smectics the fluctuational effects are related to the displacement of smectic layers (the undulational mode). Besides this, in smectics C, by virtue of their lower symmetry (in comparison with smectics A or B) the dynamic anisotropy of layers (leading to the coupling of the orientational and undulational modes) plays an important role.

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